Related topics
X-rays, Compton effect, Compton wavelength, rest energy, absorption, transmission, conservation of energy and momentum, and Bragg scattering

Principle
During this experiment, the Compton wavelength is determined indirectly with the aid of X-rays. For this purpose, X-rays are scattered on an acrylic glass block. The intensity of the scattered radiation is measured with a counter tube. Then, the Compton wavelength is determined based on the transmission behaviour and on a transmission curve that was measured beforehand.

Equipment

<table>
<thead>
<tr>
<th>Item</th>
<th>Code</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 XR 4.0 expert unit</td>
<td>09057-99</td>
</tr>
<tr>
<td>1 XR 4.0 X-ray goniometer</td>
<td>09057-10</td>
</tr>
<tr>
<td>XR 4.0 X-ray plug-in unit with a Cu X-ray tube</td>
<td>09057-50</td>
</tr>
<tr>
<td>1 XR 4.0 X-ray diaphragm tube, d = 2 mm</td>
<td>09057-02</td>
</tr>
<tr>
<td>1 XR 4.0 X-ray diaphragm tube, d = 5 mm</td>
<td>09057-03</td>
</tr>
<tr>
<td>1 XR 4.0 Counter tube, type B</td>
<td>09005-00</td>
</tr>
<tr>
<td>XR 4.0 X-ray lithium fluoride crystal, mounted in a holder</td>
<td>09056-05</td>
</tr>
<tr>
<td>1 XR 4.0 X-ray Compton attachment for the X-ray unit</td>
<td>09058-04</td>
</tr>
<tr>
<td>1 measure XR 4.0 X-ray software</td>
<td>14414-61</td>
</tr>
<tr>
<td>1 Data cable USB, plug type A/B</td>
<td>14608-00</td>
</tr>
<tr>
<td>1 Plate holder</td>
<td>02062-00</td>
</tr>
<tr>
<td>1 XR 4.0 X-ray optical bench</td>
<td>09057-18</td>
</tr>
<tr>
<td>Slide mount for optical bench, h = 30 mm</td>
<td>08286-01</td>
</tr>
<tr>
<td>Additional equipment</td>
<td></td>
</tr>
<tr>
<td>PC, Windows® XP or higher</td>
<td></td>
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</tbody>
</table>

This experiment is included in the upgrade set “XRC 4.0 X-ray characteristics”.

Fig. 1: P2541701
Compton scattering of X-rays

Tasks
1. Determine the transmission of an aluminium absorber as a function of the Bragg angle and plot it as a function of the wavelength of the radiation.
2. Measure the intensity of the radiation that is scattered at an angle of a) 60° b) 90° and c) 120° on an acrylic glass block with and without an absorber.
3. Determine the Compton wavelength of the electron based on the transmission curve.

Set-up
Connect the goniometer and the Geiger-Müller counter tube to their respective sockets in the experiment chamber (see the red markings in Fig 2). The goniometer block with the analyser crystal should be located in a position in the middle. Fasten the Geiger-Müller counter tube with its holder to the back stop of the guide rails. Do not forget to install the diaphragm in front of the counter tube.

Insert a diaphragm tube with a diameter of 2 mm into the beam outlet of the tube plug-in unit for the collimation of the X-ray beam.

For calibration: Make sure, that the correct crystal is entered in the goniometer parameters. Then, select “Menu”, “Goniometer”, “Autocalibration”. The device now determines the optimal positions of the crystal and the goniometer to each other and then the positions of the peaks.

Note
Details concerning the operation of the X-ray unit and goniometer as well as information on how to handle the monocrystals can be found in the respective operating instructions.

Procedure
- Connect the X-ray unit via the USB cable to the USB port of your computer (the correct port of the X-ray unit is marked in Fig. 3).
- Start the “measure” program. A virtual X-ray unit will be displayed on the screen.
- You can control the X-ray unit by clicking the various features on and under the virtual X-ray unit. Alternatively, you can also change the parameters at the real X-ray unit. The program will automatically adopt the settings.
Compton scattering of X-rays

Overview of the settings of the goniometer and X-ray unit for task 1:
- 2:1 coupling mode
- Gate time 100 s (gate timer); angle step width 0.1°
- Scanning range $5.5° < \vartheta < 9.5°$
- Anode voltage $U_A = 35$ kV; anode current $I_A = 1$ mA

Task 1: Determination of the transmission of aluminium
Use the analyser crystal lithium fluoride and insert a diaphragm tube with a diameter of 2 mm into the beam outlet of the tube plug-in unit for the collimation of the X-ray beam

Settings: See Overview
Determine the pulse rate $n_1(\vartheta)$ of the X-rays reflected by the crystal in angle steps of 0.1° between the glancing angle $\vartheta = (5.5–9.5)°$, by means of synchronized rotation of the crystal and the counter tube in the angular relationship 2:1. Use a measuring time of 100 s.
Repeat the measurement after you have positioned the aluminium absorber in front of the out-
let of the X-ray plug-in unit using the plate holder mounted in the slide mount on the optical bench to measure the pulse rate $n_2(\vartheta)$.

In order to keep the relative error of $n$ as small as possible, high rates are necessary.

At high pulse rates, however, the dead time $\tau$ of the counter tube must be taken into consideration since the counter tube does not register all of the incident photons (see theory and evaluation).

**Task 2: Determination of the Compton scattering**

Remove the analyser crystal and replace it with the acrylic glass scatterer. Position this at an angle of 10° (see Figs. 7 and 8). Replace the diaphragm tube with an aperture of $d = 2$ mm with the one with an aperture of $d = 5$ mm. Turn the counter tube to a) 60°, b) 90°, c) 120°.

Measure the pulse rates using the following set-ups:

$N_3$: with the acrylic glass scatterer but without the aluminium absorber

$N_4$: with the acrylic glass scatterer and with the aluminium absorber in position 1 (use the plate holder to fix it).

$N_5$: with the acrylic glass scatterer and with the aluminium absorber in position 2.

For the measurement of $N_4$, position the aluminium absorber between the diaphragm and the scatterer (use the plate holder to fix it). For the measurement of $N_5$, the aluminium absorber is fastened to the Geiger-Müller counter tube by pushing it into the diaphragm that is installed in front of the counter tube.

For every set-up, note down three measurement values. The measuring time is 100 seconds.

At very low pulse rates, it may be necessary to take the background radiation into consideration at $U_A = 0$ V.

**Theory**

The absorption of a material is determined by three different interaction processes. Their relative contributions depend on the atomic number (nuclear charge number) $Z$ and on the mass number $A$ of the shielding material.

The most important individual processes are:

- Photoelectric effect; attenuation $\sim Z^4/A$
Compton scattering of X-rays

- Compton scattering; attenuation \( \sim Z/A \)
- Pair generation; attenuation \( \sim Z^2/A \).

As a result, the energy-dependent absorption coefficient of a material \( \mu \) consists of the absorption coefficient of the pair generation \( \mu_{Pa} \), of the photoelectric effect \( \mu_{Ph} \), and of the Compton effect \( \mu_{Co} \). Two additional mechanisms, the nuclear photoelectric effect and the normal elastic scattering, can usually be neglected for the screening effect.

\[
\mu = \mu_{Pa} + \mu_{Ph} + \mu_{Co}
\]

For the X-radiation range that we focus on (\( E \approx 1\text{–}100 \text{ keV} \)), \( \mu_{Pa} \) can be neglected (see Fig. 9). \( \mu_{Ph} \) is also not relevant for this experiment since only electrons, and not photons, are released. As a result, the detector detects nearly exclusively Compton fractions.

A schematic representation of the Compton effect is shown in Fig. 10. Due to the interaction with a free electron in the solid material, the incident photon loses energy and is scattered from its original direction under the scattering angle \( \theta \). The electron that was previously at rest absorbs additional kinetic energy and leaves the collision point under the angle \( \phi \).

Based on the principle of conservation of energy and momentum, the energy of the scattered photon is obtained as a function of the scattering angle (see the appendix):

\[
E_2 = \frac{E_1}{1 + \frac{E_1}{m_0c^2} (1 - \cos \theta)} (1)
\]

Photon energy before or after the collision \( E_1 \) or \( E_2 \)
Scattering angle \( \theta \)
Speed of light in vacuum \( c = 2.998 \times 10^8 \text{ m s}^{-1} \)
Rest mass of the electron \( m_0 = 9.109 \times 10^{-31} \text{ kg} \)

After the collision, the photon has a smaller energy \( E_2 \) and, therefore, a greater wavelength \( \lambda_2 \) than before the collision. With \( E = h \nu \), (1) can be converted into:

\[
\frac{1}{h \nu_2} - \frac{1}{h \nu_1} = \frac{1}{m_0c^2} (1 - \cos \theta)
\]

Planck's constant \( h = 6.626 \times 10^{-34} \text{ Js} \)
Photon frequency \( \nu \)

With \( \lambda = c/\nu \), equation (2) leads to:
For 90°-scattering, the wavelength difference, which consists only of the three universal components, leads to the so-called Compton wavelength $\lambda_C$ for electrons.

$$\lambda_C = \frac{h}{m_e c} = \frac{6.626 \times 10^{-34} \text{ Js}}{9.109 \times 10^{-31} \cdot 2.998 \times 10^8 \text{ kg} \cdot \text{ms}^{-1}} = 2.426 \text{ pm}$$

For the special cases of backward scatter ($\vartheta = 180°$), the change in wavelength is $\Delta \lambda = 2\lambda_C$.

### Evaluation

**Task 1:** Determine the transmission of an aluminium absorber as a function of the Bragg angle and plot it as a function of the wavelength of the radiation.

Based on the glancing angles $\vartheta$ as well as on the Bragg relationship, the associated wavelengths $\lambda$ are obtained:

$$2d \sin \vartheta = n \lambda$$

with $d = 201.4 \text{ pm} = \text{LiF-(200)}$ interplanar spacing and here: $n = 1$.

For a given gate time $\Delta t$ and the pulse rate $n$ the total number of incidents $N$ is $n \cdot \Delta t$.

For the measured number of incidents $N$, the relative error of $N$ is given by the ratio:

$$\frac{\Delta N}{N} = \frac{\sqrt{N}}{N} = \frac{1}{\sqrt{N}}$$

At high pulse rates, the dead time $\tau$ of the counter tube must also be taken into consideration, since it does not register all of the incident photons. For the GM counter tube that is used in this experiment, it is 90 µs.

The true pulse rate $n^*$ can be obtained from the measured pulse rate $n$ with the aid of:

$$n^* = \frac{n}{1 - \tau n}$$

Correct the measured count rate in an angle range of $5.5° < \vartheta < 9.5°$ and with the dead time $\tau = 90 \mu s$ of the Geiger-Müller counter tube. This can be done using the software measure:

Select “analysis”, then “X-ray spectroscopy” and “dead time correction”. Now you can either create a new measurement with the corrected data or add the corrected graph into the old measurement.

The true pulse rates are then used to determine the transmission curve

$$T(\lambda) = \frac{n^*_2(\text{with an absorber})}{n^*_1(\text{without an absorber})}$$

It is then plotted as a function of $\lambda$ (see Fig. 11).

**Task 2 and 3:** Measure the intensity of the radiation that is scattered at an angle of 60°, 90° and 120° on an acrylic glass block with and without an absorber and determine the Compton wavelength of the electron based on the transmission curve.

In this experiment, the aluminium absorber acts as a kind of strong colour filter. It absorbs shorter wavelengths less strongly than longer wavelengths. This means that if it is positioned in front of the scatterer, it has a different effect than in the long-wave scattered radiation behind the scatterer. This enables the
determination of the wavelength of the scattered and non-scattered radiation.

By placing the absorber in the ray path between the X-ray tube and the scatterer (position 1, Fig. 8), you can determine the transmission $T_1 = n_3 / n_4$ of the still non-scattered X-radiation. When the absorber is in position 2, you obtain the transmission of the scattered X-radiation.

Since we have determined the dependence of the transmission of aluminium on the wavelength in task 1, we can now directly infer from the transmission to the wavelength of the X-radiation that passes through the absorber. The two different transmission coefficients that are obtained from the 90° scattering ($T_1 > T_2$) then lead to the corresponding wavelengths.

**Sample results**

Fig. 11 shows transmission curve of aluminium in a narrow wavelength range including the equation of the regression line:

$$y = -0.018x + 1.3928$$

The results from Task 2 for $n_3^*$, $n_4^*$ and $n_5^*$ are listed in table 1

**Sample calculation for the 90° scattering:**

$$T_1 = \frac{N_4^*}{N_3^*} = \frac{n_4^* \cdot \Delta t}{n_3^* \cdot \Delta t} = \frac{83.625 \text{ Imp/s}}{241.121 \text{ Imp/s}} = 0.347 \pm 1.27\%$$

$$T_2 = \frac{N_5^*}{N_3^*} = \frac{76.017 \text{ Imp/s}}{241.121 \text{ Imp/s}} = 0.315 \pm 1.32\%$$

The deviation of $\pm 1.27\%$ and $\pm 1.32\%$ is calculated with the aid of equation (2) and the following equation (use the total number of incidents $N^* = n^* \cdot \Delta t$ for the calculation):
The relative error only takes into account the statistical errors. Systematic errors (see note) are not considered.

Based on the linear equation of the regression line in Fig. 11, the associated wavelengths for the 90° scattering result as 58.93 pm and 61.48 pm. This results in a wavelength difference of $\Delta \lambda = \lambda_C = 2.56$ pm, which is very close to the theoretical value of $\lambda_C = 2.426$ pm.

Table 1 sample results

<table>
<thead>
<tr>
<th>$\vartheta$</th>
<th>$n_1^*$</th>
<th>$n_4^*$</th>
<th>$n_5^*$</th>
<th>$T_1$</th>
<th>$T_2$</th>
<th>$\lambda_1$</th>
<th>$\lambda_2$</th>
<th>$\Delta \lambda$</th>
</tr>
</thead>
<tbody>
<tr>
<td>60°</td>
<td>241.121</td>
<td>83.625</td>
<td>76.017</td>
<td>0.347</td>
<td>0.315</td>
<td>58.09</td>
<td>59.87</td>
<td>1.78</td>
</tr>
<tr>
<td>90°</td>
<td>178.833</td>
<td>59.315</td>
<td>51.235</td>
<td>0.332</td>
<td>0.286</td>
<td>58.93</td>
<td>61.48</td>
<td>2.56</td>
</tr>
<tr>
<td>120°</td>
<td>216.124</td>
<td>70.444</td>
<td>58.81</td>
<td>0.336</td>
<td>0.272</td>
<td>58.71</td>
<td>62.26</td>
<td>3.56</td>
</tr>
</tbody>
</table>

The experiment show that with decreasing scattering angle, the difference in wavelength also decreases.

**Note**
- There is a systematic error, since
  - X-rays are also diffracted at the aluminium sheet so that they might actually enter the counter directly.
  - The incident x-radiation is polychromatic
  - The geometry of the scattering region is not spherical
- The error caused by fluorescence is negligible because the corresponding radiation energy is too low to be registered.